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LETTERS TO THE EDITORS

ON METHODS OF STUDYING HEAT TRANSFER IN TRANSITION BOILING

IN THIS Journal Dr Stephan's remarks [1] were published concerning my paper [2]. I think short comments on his notes are necessary.*

1. In the first item Dr Stephan has noted that when investigating stability, the capacitive properties of the heating wall must be considered as was done in his work [3]. It may be demonstrated that capacitive properties affect the rate of change of temperature, but not the stability of the process. For checking the conditions obtained in work [2] the present author used the method of small disturbances recommended by Dr V. D. Vilensky of High Temperature Institute, Academy of Sciences of the USSR.

The temperature distribution in a flat wall ($x = 0$) heated by saturated vapour from one side ($\theta = \theta_0$) with the other side ($x = \delta$) in contact with boiling liquid ($\theta = 0$) is governed by the conditions

$$\left.\begin{aligned} &\frac{d^2\theta}{dX^2} = 0, \\ &\lambda \frac{d\theta}{dX}\bigg|_{X=0} - \alpha(\theta_0 - \theta)|_{X=0} = 0, \\ &\lambda \frac{d\theta}{dX}\bigg|_{X=1} - q(\theta)|_{X=1} = 0. \end{aligned}\right\} \quad (1)$$

where $X = x/\delta$, $\theta = t - t_s$ is the temperature difference, $q(\theta)$ is the heat flux from the wall to boiling liquid.

If at a certain instant of time ($\tau = 0$) a small disturbance is introduced, then the temperature distribution is described by a new function

$$\vartheta = \theta + \Delta\theta, \qquad \vartheta_s = \Delta\theta_s.$$

In case of small deviations we may write

$$q(\vartheta - \vartheta_s) = q(\theta + \Delta\theta - \Delta\theta_s) = q(\theta) + \frac{dq}{d\theta}(\Delta\theta - \Delta\theta_s).$$

The temperature distribution behaviour is governed by the conditions

$$\begin{aligned} &\frac{\partial\Delta\theta}{\partial Fo} = \frac{\partial^2\Delta\theta}{\partial X^2}, \\ &\frac{\partial\Delta\theta}{\partial X}\bigg|_{X=0} + Bi\,\Delta\theta|_{X=0} = 0, \\ &\frac{\partial\Delta\theta}{\partial X}\bigg|_{X=1} - \psi\Delta\theta|_{X=1} = -\psi\Delta\theta_s, \end{aligned} \quad (2)$$

where

$$\psi = \frac{dq}{d\theta}\cdot\frac{\delta}{\lambda}, \qquad Bi = \frac{\alpha\delta}{\lambda}, \qquad Fo = \frac{\alpha\tau}{\delta^2}.$$

The solution of equation (2) may be expressed in a series form [4]

$$\Delta\theta = \sum_{n=1}^{\infty} A_n\varphi_n \exp(-\beta_n Fo)$$

where φ_n are eigenfunctions to be determined from the conditions

$$\begin{aligned} &\varphi'' + \beta^2\varphi = 0, \\ &\frac{\partial\varphi}{\partial X}\bigg|_{X=0} + Bi\varphi|_{X=0} = 0 \\ &\frac{\partial\varphi}{\partial X}\bigg|_{X=1} - \psi\varphi|_{X=1} = 0. \end{aligned} \quad (3)$$

In case of stable heat transfer, condition $\beta_n > 0$ should be satisfied. Stability will be disturbed if $\beta_1 = 0$. From (3)

$$\varphi_1'' = 0, \qquad \varphi_1 = AX + B$$

can be obtained. Substitution of φ_1 into (3) yields

$$\frac{1}{1/\alpha + \delta/\lambda} = -\frac{dq}{d\theta},$$

which is the result obtained in [2]. Thus, heat capacity of the heating wall does not enter into the condition of heat transfer stability.

2. The second comment of Dr Stephan is not clear since in the paper it is mentioned in two places (pp. 281 and 283) that the region of minimum (but not maximum) value of the derivative $dq/d\theta$ is the region most difficult to investigate.

The analysis of the data in work [2] has allowed the present author to infer that the temperature driving force θ_m corresponding to a minimum value of the derivative

* Work [2] was carried out at the end of 1965.

$dq/d\theta$ is close to θ_{cr_1}. This is confirmed by the data of work [5]: $\theta_m - \theta_{cr_1} \simeq 40 - 27 = 13°C$, $\theta_{cr_2} - \theta_m \simeq 150 - 40 = 110°C$.

3. The method suggested by Dr Stephan is known [6]. However the present author's [2] is still useful. First, at a horizontal plate the conditions are readily obtained when all the points of the surface are at strictly equal conditions and at equal temperature. Second, the procedure which uses condensing vapour as the heating medium is relatively simple and therefore is widely used for investigation of transient boiling heat transfer.

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REFERENCES

1. K. Stephan, On methods of studying heat transfer in transition boiling, *Int. J. Heat Mass Transfer* **11**, 1735–1736 (1968).
2. S. A. Kovalev, On methods of studying heat transfer in transition boiling, *Int. J. Heat Mass Transfer* **11**, 279–283 (1968).
3. K. Stephan, Stabilität beim Sieden, *Brennst.-Wärme-Kraft* **17** (12), 571–578 (1965).
4. A. V. Luikov, *Analytical Heat Diffusion Theory*. Academic Press, New York (1968).
5. S. Ishigai and T. Kuno, Experimental study of transition boiling on vertical wall in open vessel, *Bull. JSME* **9** (34), 361–368 (1966).
6. P. G. Poletavkin, V. I. Petrov, L. D. Dodonov and I. T. Aladiev, A new method of investigation of boiling heat transfer, *Dokl. Akad. Nauk SSSR* **40**, 5 (1953).

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NOTE ON THERMAL INSTABILITY OF A HORIZONTAL LAYER OF NON-NEWTONIAN FLUID HEATED FROM BELOW

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In a recent paper Tien *et al.* [1] considered the convective stability of a horizontal layer of a power-law fluid which is heated from below. They used an approximate theoretical analysis to derive conditions for stability. Apparently, these authors were not aware of the recent linearized stability analysis by Green [2] and by Vest and Arpaci [3]. Green used an Oldroyd constitutive equation and found that for fluids that are not extremely elastic the onset of motion occurs at the same Rayleigh number as for a Newtonian fluid. Vest and Arpaci obtained the same result for a Maxwell model. These results are reasonable since both models reduce to a Newtonian fluid in the limit of zero shear. The power-law model used by Tien *et al.* does not reduce to a Newtonian fluid in the limit of zero shear and thus is not the best choice of constitutive equation to study the onset of convective motion.

The stability analysis used by Tien *et al.* is closely related to the energy method of stability analysis and should be useful with other constitutive equations. The energy method was first applied to convective stability problems by Joseph [4] and to the stability of non-Newtonian fluids by Feinberg [5]. This method is closely related to Liapunov stability methods [6]. Since the development of the stability equations is considered in detail elsewhere [4–6], only a brief outline will be given.

The equations of motion and heat transfer are written for a basic flow $\bar{V}$, $\bar{\tau}$, $\bar{T}$, $\bar{p}$ and a disturbed flow $\bar{V}^*$, $\bar{\tau}^*$, T^*, p^*. The Boussinesq approximation is assumed. Define the perturbations,

$$\boldsymbol{u} = \boldsymbol{V}^* - \bar{\boldsymbol{V}}, \qquad \Delta\tau = \tau^* - \bar{\tau}, \qquad \theta = T^* - \bar{T} \quad (1A, B, C)$$

and subtract the basic flow from the disturbed flow. After forming the scalar product of $\boldsymbol{u}$ with the equations of motion and of θ with the heat-transfer equation, integrate the results over the flow volume V. This volume extends from $z = 0$ to $z = d$ and over a period of the perturbations in the x and y directions. The result is [4, 5]

$$\frac{dK}{dt} = \int_V \left[\frac{1}{\rho}\boldsymbol{u}\cdot\nabla\cdot\Delta\tau - \boldsymbol{u}\cdot\boldsymbol{D}\cdot\boldsymbol{u} - \alpha\theta\boldsymbol{g}\cdot\boldsymbol{u}\right] \times dV + \int_S \frac{p^* - \bar{p}}{\rho}\boldsymbol{u}\cdot\boldsymbol{N}\,dS \quad (2)$$